Detection of Underwater Moving Object 
Based on the Compressed Sensing

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Motivation
To detect the moving state of underwater vehicles real-time and accurately has been an interesting topic. And linear frequency modulation signal is influenced by different motion state of the target under test, and its echo parameters, such as the initial frequency, frequency modulation rate, phase, will have corresponding change according to the target motion state. So we propose a detection method about motion state of underwater target based on compression sensing.

Sonar Echo Model

A. echo signal of stationary target
\[ s_a(t) = A \text{rect} \left( \frac{t}{b} \right) \]
\[ \exp \left\{ j 2 \pi \left[ f_c (t - \tau_0) + \frac{1}{2} k (t - \tau_0)^2 \right] \right\} + n(t) \]

B. echo signal of uniform motion target
\[ s_a(t) = A \text{rect} \left( \frac{t}{b} \right) \]
\[ \exp \left\{ j 2 \pi \left[ f_c (t - \tau_0) + \frac{1}{2} k (t - \tau_0)^2 \right] \right\} + n(t) \]

C. echo signal of uniformly variable motion target
\[ s_a(t) = A \text{rect} \left( \frac{t}{b} \right) \]
\[ \exp \left\{ j 2 \pi \left[ f_c (t - \tau_0) - \frac{2 \pi f_c}{c} t - \frac{2 R_c}{c} \left[ a(t) \right]^2 + \frac{1}{2} k r^2 \right] \right\} \]

D. echo signal of variable accelerated motion target
\[ s_a(t) = \exp \left\{ j 2 \pi \left[ f_c (t - \tau_0) - \frac{2 \pi f_c}{c} t - \frac{2 R_c}{c} \left[ a(t) \right]^2 + \frac{1}{2} k r^2 \right] \right\} + n(t) \]

Detection and Recognition
We proposed a high order Chirplet Transform whose kernel function has certain bending effects. The high order Chirplet Transform is shown as follow,
\[ h(t-t',f,a,c) = \frac{1}{\sqrt{2 \pi a \sigma}} \exp \left\{ \frac{1}{2} \left( \frac{t-t'}{\sigma} \right)^2 \right\} \]
\[ \exp \left\{ j \pi \left[ f(t-t') + c(t-t') + \frac{k}{a} (t-t')^2 \right] \right\} \]

According to the convex optimization theory, we can obtain the compressed sensing as follow,
\[ \min \left\| y - \Psi \Phi \right\|_2^2 + \lambda \left\| \Psi \right\|_1 \]

The coordinates of the vertical axis can represent the order of echo signal of LFM: if the end point of the curve in the interval [1600,1800], the signal is first-order chirp echo signal; if the end point of the curve in the interval [1400,1600], the signal is second-order chirp echo signal; if the end point of the curve in the interval [1200,1400], the signal is third-order chirp echo signal; and other and so on.

CONCLUSION
In the above, the experimental results show that compared with the Short Time Fourier Transform(STFT), Wavelet Transform(WT), Winger-Ville Distribution or other time-frequency analysis method, the higher order Chirplet Transform has the very high resolution without cross-term inference, and it is suitable for analyzing the non-stationary signal underwater acoustic. After obtaining characteristics of the time-frequency of echo signal, the main characteristics of the data are extracted by the compressed sensing based on the Noiselets matrix, and the interference from the underwater acoustic channel noise is eliminated.